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PHYSICS—*A tree from the viewpoint of lightning.*[1]

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Trees are occasionally struck by lightning when they add salient features to the landscape. The results of lightning strokes vary from subsequent death of all or a portion of a tree to a tearing of the bark which frequently heals and in some cases heals so well that after a few years there is little visible evidence of the injury. Occasionally the heart wood of a tree trunk is blown apart as the tree is shattered or demolished, whereas more often narrow sections of the bark are stripped off with but slight damage to the heart-wood section. The blasting of bark from a tree by lightning has been ascribed to a stream of rainwater that runs down one side of the trunk, provides a conducting path, and is converted by the lightning current to steam that blasts the bark off the tree. However, as Plummer (*1*) reports, the scored path frequently follows the grain of the wood. This often results in a spiral groove where the bark is blown off the spiraling grain of the outer sapwood. This and the fact that sap-wood is frequently shredded into rather small fibers, suggested that the wood moistened by its own sap might initially provide a much better conductor than a stream of rain water—nature's distilled water—outside the bark. Thus the author was led to doubt that the stream of water outside the bark plays any important role in damage to trees from lightning.

An unequivocal statement appears in National Bureau of Standards Handbook 46 under the section on Personal Conduct, in the Code for Protection Against Lightning: "If remaining out of doors is unavoidable, *keep away from isolated trees.*" Before sharing with others any differing viewpoint, because of the hazards involved, the author felt obliged to find in the literature or to determine experimentally the electrical resistance characteristics of a tree and its root connection to earth. A library search did not yield any values so that representative electrical resistance measurements reported elsewhere were made on a few trees. For the purpose of this discussion only the results presented in Fig. 1 are of particular interest. This figure was synthesized from the measurements made on parts of a green tulip tree—leaves, twigs, branches, trunks, and from the trunk to a pipe of a water system some 80 meters away that served as the electrical "ground" for these measurements.

Several interesting resistance features of this synthetic tulip "tree" should be mentioned. The resistance measurements were made in such a manner that the values correspond to charge flowing to ground as in a lightning stroke to a tree. As one moves radially toward the tree from a good "ground" connection the resistance increases rapidly as shown by the ohms to "ground" values indicated on the right side and up the side of the "tree" in Fig. 1. Values increase still more rapidly going up the trunk of the tree and on out to the tip end of a leaf on a twig. Actually the resistance from the tip end of a leaf to its supporting twig is about 6 times as large as the resistance from the tip end of a twig to the base of the tree, whereas the resistance from the tip end of a twig to

[1] An abridgement of a talk given on February 17, 1955, by the author as retiring president of the Washington Academy of Sciences. The technical details presented on the measurements of the electrical resistance to earth of a live tree are being submitted for publication elsewhere.

the base of the tree trunk is about 1,000 times as large as the resistance from the base of the trunk to "ground." Another measurement should be mentioned. Sample sections of a tulip branch were found to have a rather high negative temperature coefficient of resistance. The resistance decreased by a factor of about 10 when the temperature was raised from 20° to 75°C.

DISCHARGES FROM A TREE

First let us consider the bearing of this electrical resistance picture of a tree on pre-lightning currents, i.e., before a truly disruptive discharge occurs. (Lightning is defined as a disruptive electrical discharge between clouds or between clouds and earth.)

In view of Schonland's measurements (*2*) on the approach of a thundercloud, that gave a discharge current of 4 microamperes for an insulated thorn tree 4 m tall, our synthesized tree 25 m tall would probably discharge a current 10 times as large, if we bias our multiplying factor in the direction of extrapolated values of current for point-to-plane laboratory measurements. For this estimate of 40 microamperes the total voltage drop from ground up the trunk and out to the tip end of the leaves on the uppermost branch of our synthesized tree would only be about 10 volts.

If we consider the modification in the natural electric field over flat ground that would be produced by a tree, having more or less of a spherical outline, we can rely on the simple electrostatic case analyzed by Lord Kelvin as a guide to the modifications of the plane equipotential surfaces we might normally expect. Kelvin showed that for such a uniform field a simple smooth hemispherical metal boss mounted on a conducting plane would alter the equipotential surfaces so that the voltage-gradient directly above the boss would be three times as large as the original field strength. Actually but few trees present a top outline as stumpy as a hemispherical boss but rather they can be better represented in outline by a cone or by a smaller sphere mounted on a stem of a height that may be as large as several times

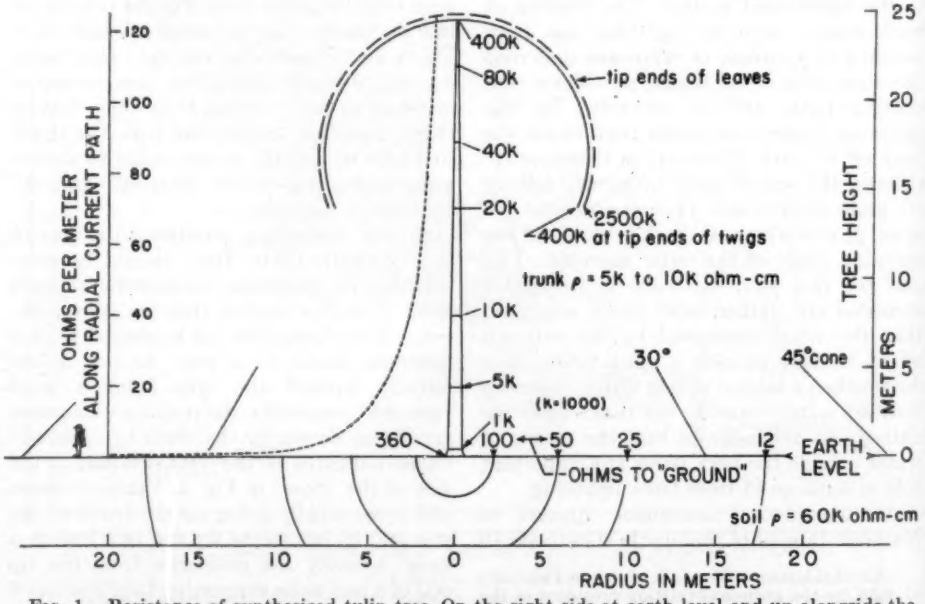

FIG. 1.—Resistance of synthesized tulip tree. On the right side at earth level and up alongside the trunk, values are marked to show the ohms to "ground." Values are also given out to the tip end of a twig and beyond to the tip end of a leaf on a twig. On the left side of the trunk a dashed line has been plotted to show ohms per meter along a radial current path based on the measurements of ohms to "ground" made at various radial distances from a tree with the trunk as one electrode connection.

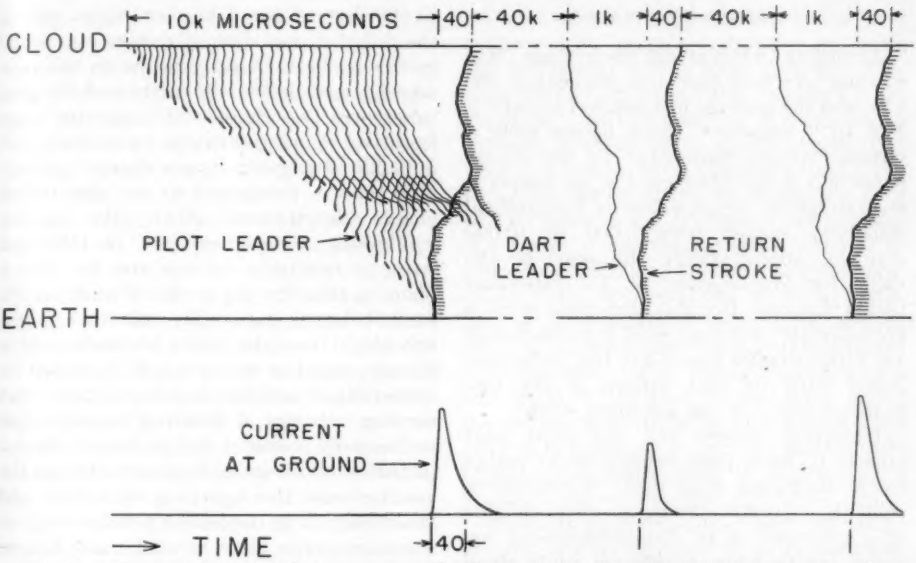

FIG. 2.—Diagram (*2*, *6*) of a three-component lightning stroke. Once the pilot leader or tip (mentioned in the text) at the end of the streamer leader reaches ground the brilliant return stroke (lightning stroke) returns to the cloud along the same path. In a multiple-component stroke the same tortuous path is renewed by one or more dart leaders with subsequent brilliant and noisy return strokes.

the diameter of the sphere. Such an outline would lead to a further increase in gradient, say by a factor of two or three. In addition to this still another multiplying factor should be introduced because of the sharp outlines and edges of the leaves in lieu of the smooth spherical conducting surface that Kelvin postulated for his simple case of the smooth hemispherical boss. These three simple considerations lead us to an overall factor much larger than three. Whether the overall factor could be stretched say from 3 to the order of 20 or 30 and thus be large enough for multiplying Schonland's measured gradient value of 160 v/cm at ground for an advancing thundercloud to obtain 3,400 v/cm, the value measured in a cloud just prior to a stroke to a plane (*3*), or to obtain 5,000 v/cm, the corresponding average breakdown gradient measured for a long laboratory spark gap (*4*), remains problematical.

Any such increase in voltage gradient near the leaves, however, should lead to enhanced ionization and to electrification of dust that might be blown past leaves in the tree top by winds.

The leaves with their high resistance, 1 to 5 megohms, would tend to share and to divide the current of the discharge into much smaller values. Thus the treetop presents the multiple electrode effect of many needles each with a resistor of several megohms in series. It would seem that discharges from a lone tree would be augmented and that a tree with its top glowing in the dark from discharge as a thundercloud advanced toward and over it should be reported more often. I have never seen such a sight at night, my only experience being limited to seeing in daylight a tall poplar tree with its top leaves agitated violently, as if by a vertical wind with no circular motion. This occurred early one afternoon when such a glow would not have been observable. Presumably this was a case of a strong vertical electric wind because leaves on nearby lower trees were not in motion. It was otherwise very calm with thunderclouds about a half-mile distant. This effect persisted for several minutes then stopped.

STROKES TO A TREE

According to Schonland the average "fine weather" vertical electrical gradient is 100 v/m and the average fine weather current is 2×10^{-16} amp/cm^2. These figures yield a virtual "volume resistivity" for the atmosphere of 5×10^{17} ohm-cm. For the trunks of several "green" trees, values of longitudinal resistivity ranged from 5,000 to 10,000 ohm-cm, approximately the same as the value for Washington, D. C., tap-water (5,000 ohm-cm). Roughly then green wood is normally 10^{13} times as good a conductor as air. Little wonder that a live tree is the preferred path for the current if the tree is within reach of the lightning stroke.

Lightning strokes for the most part appear to have their beginnings as rather feeble discharges in or near "thunderstorm cells" in cumulus clouds. At cloud heights of a kilometer or so thunderstorm-cell activity begins when a thickish white cloud (cumulo-nimbus) begins to well up in the middle and tower to greater heights. Moisture-laden air is forced in at the visible cloud base and loses part of its moisture through the condensing of additional small droplets that help form the cloud base. Their heat of condensation warms the moisture-depleted air causing it to rise even higher into cooler surroundings where additional moisture is lost. Eventually, robbed of its moisture except for fine ice crystals or tiny droplets buoyed up by it, this air spreads the particles (charged and neutral) out to form the well-known anvil-shaped top of the cumulo-nimbus cloud at some 10 to 15 kilometers altitude. This process constitutes the basis for separating large quantities of electric charges—of separating electrons from ice particles or droplets that are left positively charged. Many of the electrons are undoubtedly captured by larger falling droplets and form negative ions.

Without going into the rather well substantiated explanations (*2*) of the intimate processes of charge separation we may look at the end results. It is well confirmed, by observations that generally, but not always, the lightning discharge transports negative charge from the cloud to ground. The usual arrangement then is a predominately negatively charged cloud base or region say of the order of one kilometer or so above the earth's surface. Directly below on the conducting surface of the earth and its protuberances a charge of opposite sign (positive) is induced that is equal to the net unbalanced negative space charge immediately above. Compared to the gust-ruffled charge distribution above, the earth's conducting surface, in spite of hills and trees, is relatively smooth and flat like a plane so that the charge distribution on the earth below is reasonably uniform. Indeed one might consider the undersurface of a thunder cloud as comprised of connected inverted dome- and pinnacle-like surfaces that envelop volumes of differing negative (an occasionally positive) charge density. Such a picture would seem to help us to bridge the gap between the lightning of nature and laboratory surge discharges possible only on a dimensionally much smaller scale. Except for very suddenly applied high overvoltages (voltages considerably higher than normally required to complete electric breakdown for a voltage applied gradually) the initial discharges from negative points toward planes are of a long filamentary character.

The filamentary beginnings of a lightning discharge should be thought of as originating (near the cloud) where the electric gradient is largest between rather highly charged pockets or regions of separated charges. At the expense of this electric field (born of the bulk mechanical separation of charges), some of the charges are accelerated to velocities capable of producing copious ionization. Then a lower voltage gradient suffices to maintain ionization as well as to transport charge for a brief interval along the "conducting" channel or filament, until the recombination of ions reduces conductivity. Thus for a brief interval the voltage gradient is augmented at either end of the filament and leads to an energetic momentary extension of the conducting filament toward the centers of the charged regions or toward the induced charges at the surface of the earth. As recombination depletes the carriers the current or transport of charge decreases and the expanded channel requires a larger gradient throughout its length to restore the ionization.

Thus the chain of events leading to lightning striking a tree is a hit or miss affair in which the tree contributes very little if anything in the initial stages. High-speed photography of strokes (*2*) reveals the stepwise manner in which a faintly luminous discharge beginning at the cloud end of a stroke halts in its progress toward the induced charge on the earth's surface. Fig. 2, derived from Schonland's photographs, is a graphic illustration of a three-component stroke along the same path. Each luminous step of some 30 to 50 meters length formed at the tip end of the fainly luminous leader is formed rapidly in about one microsecond then fades until the accumulated path of the leader from the source brightens again as the tip again rushes forward for another microsecond or so revealing by its brilliance that considerable energy had been used in its formation. One interesting feature about these steps is their frequent change of direction after a rest or pause of about 50 microseconds. This change is presumably indicative of the changing direction of the highest gradient or the most readily ionizable path. From what has been said earlier one should not expect, even under the most favorable situation, a streamer to project upward to any considerable height from a tree to meet an advancing stepped leader. Only if the last two or three steps lie in its direction may a tree be expected to contribute a slight directional effect because of the higher-than-average voltage gradient existing above the tree.

THE "TREE" AS A LIGHTNING SHELTER

Figure 1 that shows the synthesized electrical-resistance picture of a tree, has several features added to help visualize how dangerous it is to use a tree as a shelter from rain and lightning at the same time. We humans are better conductors of electricity than trees. Our skins offer poorer insulation than the bark of trees either wet or dry because of numerous pores and the saltiness of perspiration. Our resistance from one foot to the other[2]—omitting skin—is roughly 150 ohms, from both hands to both feet 370 ohms, whereas our trunk section considered as a four-terminal resistor measured by using the hands and feet is only 25 ohms. This suggests two rather obvious reasons for not standing close to the trunk of a tree in an exposed location in a thunderstorm, i.e., close to a tree standing alone in a field, a tall tree that towers above its neighbors, or a tree on top of a hill or ridge: 1) In the case of a stroke to the tree one would be likely to receive a sideflash to his body and thus provide a path for part of the stroke to earth. This is understandable because the median value of lightning currents is 20,000 amperes crest and even for an *initial* resistance of a tree trunk of 100 to 1,000 ohms per meter the voltage drop might attain 4 million volts or more in 6 feet, the height of a man, and 2) because the radial drop in resistance to ground is highest near the trunk.

We must examine more closely the values of resistance to "ground" mentioned earlier in connection with Figure 1. These values on the right side of the "tree" have been replotted as a dashed curve on the left side. This curve is advantageous in showing how rapidly, r_f, the radial resistance per meter drops with distance away from the tree trunk. The abscissae of this plot is the radius in meters and the ordinates the ohms per meter along radial current path. This curve of r_f provides an index of the danger from this source of voltage that could send lethal current up one leg and down the other.

The simple expression $E = I\, r_f\, f \cos\theta$ is indicative of this danger, where E is the voltage drop between the two feet at a distance of f apart in meters; r_f is the radial resistance per meter where the feet are located; I is the stroke current and θ is the angle between a line through the two contacts of the feet with ground and the radius line to the stroke terminus. Thus the voltage, E, is available to puncture the shoes and the skin on the feet and to force current through the resistance offered by the rest of the bystander's body.

There appears to be some substantiation of the theoretical cos θ relationship. Among others, an instance was reported about 15 years ago of a stroke to a lone tree in a pas-

[2] Private communication from F. Wenner and Irvin L. Cooter, of the National Bureau of Standards.

ture at the Beltsville farm in which 6 cows that had taken shelter under the tree were killed. These cows were found dead with their fore and hind hooves pretty well aligned radially with the struck tree whereas several other cows under the tree were not killed by the stroke and presumably had been standing in a tangential position at the moment of the discharge. We humans are lucky because we have only two feet to worry about. It appears from the equation that if we were able to stand with our feet in line circumferentially with the base of the stroke as a center, then θ would be 90° and cos θ would be zero. Thus we should be relatively safe. One way to make the voltage E between our feet a minimum would be to reduce f, the distance on the ground between our two feet. The simplest way to do this would appear to be to stand on one foot till it gets tired and then on the other! It is also more or less obvious that a bystander should not remain under low-lying branches that would provide a rather short air gap between the branches and his head because of the good conducting path his body provides to earth. We can do nothing about the current I. We have to accept it at its average value of 20,000 amperes crest if we have just average luck. We should probably "play safe" at this point and assume a stroke of 100,000 amperes. For this value of current a radial resistance per meter of ⅓ ohm would give a voltage drop of 30,000 V per meter. If our feet were ⅓ m apart the voltage drop between them would correspond to 10,000 V. This value of voltage might produce uncomfortable results, but I rather doubt that it would prove lethal. From our measurements (Fig. 1) ⅓ ohm per meter corresponds to a distance out from the tree trunk of 12 meters. Undoubtedly the resistance per meter continues to fall beyond 12 meters, the most remote of the radial measurements made. Would we not be safer at a distance considerably farther away from the trunk?

The instructions quoted earlier said "keep away from isolated trees." Fig. 1 and what has just been said clearly support the advisability of keeping at a safe distance. How far? Herein lies the paradox for there is another consideration—the shielding effect which tells us not to move *too far* away from a tall exposed tree. A well-grounded conducting mast (a tree is similar) of height h partially shields that space adjacent to the mast so that it is relatively immune from direct strokes of lightning. For a cone centered on the mast of height h and having a base of diameter h, laboratory measurements indicate that out of 1,000 strokes, the mast would on the average receive 999 strokes for every single stroke received by a second conducting rod one-third as tall as the mast if located just within the base of the cone. The angle for this case where $r = 0.5h$ would represent a "30° cone of protection." This chance of 1 in 1000 of being struck is referred to as an exposure of 0.1 percent for the rod (or object) relative to its protecting mast. Actually the protected rod 8 m tall would be 4 times the height of a tall man. For a person 2 m (6½ feet) tall the exposure to direct stroke under our synthesized tree 25 m tall appears from laboratory tests[5] to be almost nil within a 30° cone of protection. It is much less severe than 1 in 1200 out to the edge of a 45° cone of protection.

The frequency with which isolated masts less than 500 feet high are struck varies directly with their height (*6*). From this it would appear that our synthesized tree 25 m tall, from height considerations alone, would be about 12 times more likely to be struck than a person 2 m tall under conditions of similar exposure. That is by moving just within the 45° cone of protection of the tree the bystander moves into the tree cone-subtended area which is 12 times more likely to be struck than is his own cone-subtended area if he remains in the open, but he gains by a factor (reciprocal of exposure) of well over 1,200 in being less likely to receive a direct stroke. His net gain from the shielding effect of the tree could thus be represented by an "improvement factor" of at least 100 (1,200 divided by 12) although he takes chances on sharing more of the stroke current that may pass in one foot and out the other than if he were farther away.

By moving out as far as the radius, $r = 2h$, for a 60° cone of protection, the exposure would still not become as high as 1 percent so that the improvement factor would surely not be reduced as low as 10. This

reduction in improvement factor from 100 to 10 dependent on whether the bystander remains just within the 45°—or moves out so as to be just within the 60° cone of protection should make us aware that the reduction in susceptibility to ground current sought by increasing radial distance from a tree may be largely offset by increased exposure. By moving too far away from the tree the bystander may move closer to the base of those nearby strokes that miss both him and the tree and thus he would gain in the likelihood of sharing the ground current from other strokes.

In conclusion then, from the viewpoint of lightning a tree initially offers a better path to earth. Lightning, however, is quite near-sighted and a good tree may be hard for lightning to find unless it stands in the open. In such a case a good way to hide from lightning is to "stand well out from under" the branches of a tree. It would appear that the Handbook 46 admonition: "*Keep* away from isolated trees in a thunderstorm," has a rather good justification. Because of the paradox cited, if one has no other choice than remaining in the open and there is an isolated tree 50 feet or more tall one should not mind getting wet but should "stand well out from under", but not too far, say never closer than 50 feet to the trunk, so that he may be afforded some degree of protection from lightning by the tree.

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MATHEMATICS—*An algebraic proof of the isoperimetric inequality for polygons.*[1]

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1. The problem of determining among all plane polygons P with n sides and total perimeter L, that one which encloses the greatest area F, is a classical one. It is well known that the extremal polygon is the regular one. This result can be stated as the inequality:

$$L^2 - 4n \tan(\pi/n) \cdot F \geqslant 0$$

with equality if and only if P is regular. The corresponding result for curves is

$$L^2 - 4\pi F \geqslant 0,$$

with equality if and only if the curve is a circle.

We shall be concerned with the discrete problem only; modern accounts of the subject have been given by L. Fejes Tóth [1] and T. Bonnesen and W. Fenchel [6]. Among the proofs of the isoperimetric inequality for polygons are some which are analytic (e.g. [2]) and some which are geometric (e.g. [3]). We present here an essentially algebraic proof, based on the extremal properties of the characteristic values of Hermitian matrices.[4] This proof can be regarded as a variation on Blaschke's proof ([2], p. 13), in so far as the Fourier apparatus he uses explicitly is used here implicitly.

2. For completeness' sake we sketch the reduction of the general problem to the convex equilateral case. (Cf. Courant and Hilbert, [9, p. 160]; see also Polýa and Szegö [8]).

(a) First, a non-convex polygon cannot be extremal. For if there was a non-convex part such as $\cdots ZABCD \cdots$ then we could obtain an isoperimetric polygon with a

[1] This work was supported in part by the Office of Naval Research.
[2] University of Notre Dame, American University and National Bureau of Standards.
[3] National Bureau of Standards.

[4] We have recently [7] used this method to handle inequalities which are closely related to the isoperimetric ones.

larger area by constructing $\cdots ZAB'CD \cdots$ where B' is the reflection of B in AC. (See Fig. 1.)

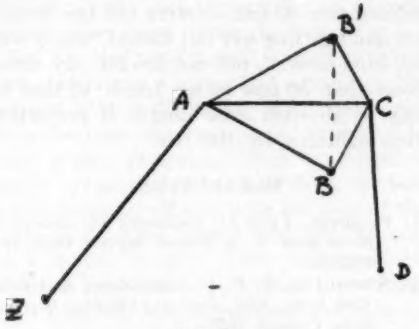

FIG. 1

Secondly, we show that a nonequilateral polygon cannot be extremal. Suppose P nonequilateral; then among the sides unequal to $s = L/n$, some will exceed s and some be exceeded by s. We shall show how to obtain an isoperimetric polygon with a larger area, and one more side equal to s. There are two cases:

(b) Suppose two adjacent sides $AB = c$, $BC = a$ satisfy $c > s > a$. In this case we can immediately replace the polygon P: $\cdots ZABCD \cdots$ by the isoperimetric polygon P': $\cdots ZAB'CD \cdots$ where $AB' = s$, $B'C = a + c - s$. The area of P' exceeds that of P because the area of $AB'C$ exceeds that of ABC. Indeed, if σ is the common semiperimeter of ABC, $AB'C$ we have

$$\sqrt{\{\sigma(\sigma - a)(\sigma - b)(\sigma - c)\}} < \sqrt{\{\sigma(\sigma - (a + c - s))(\sigma - b)(\sigma - s)\}}$$

for

$$\{\sigma - (a + c - s)\}(\sigma - s) = (\sigma - a)(\sigma - c) + (c - s)(s - a)$$

and the last term is positive because

$$c > s > a.$$

This argument may fail if P' is not convex: in this case reflections of type (a) will produce a new P'' with a larger area, to which this argument can be applied.

(c) The case when no two adjacent sides bracket the mean can be reduced to (a) by the following device. We can transpose two sides AB, BC of a polygon P: $\cdots ZABCD \cdots$ into the sides $AB' = BC$, $B'C = AB$ of the isoperimetric polygon P': $\cdots ZAB'CD \cdots$ by reflection in the perpendicular bisector l, of AC. (See Fig. 2.) The areas of P' and P coincide; if P' is not convex we can obtain a new P'' with an area greater than that of P', and with the sides in the new order, by a reflection of type (a). A finite number of repetitions of this transposition will bring the sides which bracket the mean together, and then the transformation (b) will apply.

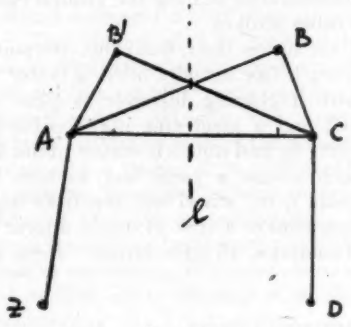

FIG. 2

After at most $(n - 1)$ repetitions of (b), we obtain a convex equilateral polygon. It remains to show that among all such polygons there is one which has the greatest area and this is the regular polygon.

3. *Theorem. If $z_1, z_2, \cdots, z_n$ are any $n(>3)$ complex numbers, and if $z_{n+1} = z_1$, then*

$$\sum_{j=1}^{n} |z_j - z_{j+1}|^2 > 2 \tan \frac{\pi}{n} \mathcal{I} \sum_{j=1}^{n} \bar{z}_j z_{j+1}, \tag{1}$$

with equality if and only if

$$z_j = \alpha \exp(2\pi i j/n) + \beta, \quad (1 \leqslant j \leqslant n) \tag{2}$$

where α, β are two arbitrary complex numbers.

Proof: Corresponding to the Hermitian form

$$\sum_{j=1}^{n} |z_j - z_{j+1}|^2 = 2 \sum_{j=1}^{n} |z_j|^2 - \sum_{j=1}^{n} z_j \bar{z}_{j+1} - \sum_{j=1}^{n} \bar{z}_j z_{j+1},$$

consider the Hermitian matrix of order n:

$$A = \begin{Vmatrix} 2 & -1 & 0 & 0 & \cdots & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & \cdots & 0 & -1 & 2 \end{Vmatrix}.$$

Corresponding to the Hermitian form

$$2\mathcal{J} \sum_{j=1}^{n} \bar{z}_j z_{j+1} = i \sum_{j=1}^{n} (z_j \bar{z}_{j+1} - \bar{z}_j z_{j+1}),$$

consider the Hermitian matrix of order n:

$$B = \begin{Vmatrix} 0 & -i & 0 & 0 & \cdots & 0 & 0 & i \\ i & 0 & -i & 0 & \cdots & 0 & 0 & 0 \\ 0 & i & 0 & -i & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & i & 0 & -i \\ -i & 0 & 0 & 0 & \cdots & 0 & i & 0 \end{Vmatrix}.$$

If we introduce the Hermitian matrix

$$H = A - \tan\frac{\pi}{n} \cdot B$$

and $v = \{z_1, z_2, \cdots, z_n\}$, then (1) becomes

$$(Hv, v) \geqslant 0. \tag{3}$$

The matrices A,B,H are circulant matrices, i.e. they are of the form $D = (d_{jk})$ where $d_{jk} = \delta_{j-k+1}$ and where subscripts on δ are to be understood mod n. The matrix D has characteristic values

$$\lambda_j = \sum_{k=1}^{n} \delta_k \exp(-2\pi ijk/n), \qquad j = 1, 2, \cdots, n$$

with corresponding characteristic vectors

$$u_j = \{1, \exp(-2\pi ij/n), \exp(-4\pi ij/n), \cdots, \exp(-2\pi i(n-1)j/n)\}.$$

These facts can be established (cf. [4], [5]) by observing that D is a polynomial in the matrix

$$C = \begin{Vmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{Vmatrix}$$

which represents the cyclic permutation of order n. In fact

$$D = \sum_{k=1}^{n} \delta_k C^{k-1}.$$

In the case of H the elements δ_k are $2, -1 + i\tan\pi/n, 0, \cdots, 0, -1 - i\tan\pi/n$ and the characteristic value λ_j of H corresponding to the characteristic vector u_j is

$$\lambda_j = 2\left(1 - \cos\frac{2j\pi}{n} + \tan\frac{\pi}{n}\sin\frac{2j\pi}{n}\right) \qquad (1 \leq j \leq n).$$

A brief verification shows that

$$\lambda_j > 0 \quad \text{for} \quad 1 \leqslant j \leqslant n-2 \quad \text{and} \quad \lambda_{n-1} = \lambda_n = 0.$$

It follows that (3) holds for any vector $v = \{z_1, z_2, \cdots, z_n\}$ in the unitary n-space. The equality sign in (3) holds if and only if v is a linear combination of the two characteristic vectors u_{n-1} and u_n, i.e., the z_i's are of the form (2).

4. The isoperimetric property for equilateral polygons is an immediate consequence of the theorem. For if the (complex) numbers $z_1, z_2, \cdots, z_n$ are the vertices of a polygon then

$$\tfrac{1}{2}\mathcal{J} \sum_{j=1}^{n} \bar{z}_j z_{j+1}$$

is its area; if the polygon is equilateral then

$$n \sum_{j=1}^{n} |z_j - z_{j+1}|^2$$

is the square of its perimeter. Our theorem shows that

$$L^2 > 4n \tan\frac{\pi}{n} F$$

unless (2) holds and this is equivalent to saying that the polygon is a regular one inscribed in the circle with centre β and radius $|\alpha|$.

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PALEONTOLOGY.—Stacheoides, *a new foraminiferal genus from the British Upper Paleozoic*. ROBERT H. CUMMINGS, University of Glasgow, Scotland. (Communicated by Alfred R. Loeblich, Jr.)

(Received August 26, 1955)

Within the foraminiferal assemblages of the Carboniferous there occur a large number of forms adapted to a fixed mode of life. Indeed, it would seem that the proportion of such forms is greater in Upper Paleozoic microfaunas than it is in those of other geological periods, and in modern assemblages.

This is particularly true in the calcareous facies of the British Lower Carboniferous, where ecological conditions in the Upper Avonian appear to have been conducive both to the attainment and development of the sessile mode of life. Representatives of such genera as *Nubecularia* Defrance, 1825, *Stacheia* Brady, 1876, *Apterrinella* Cushman and Waters, 1928, *Calcitornella* Cushman and Waters, 1928, and *Fourstonella* Cummings, 1955, are widespread and abundant in such British Carboniferous deposits.

There are some indications that the attached mode of life should be regarded as original in the more primitive foraminiferal groups, such as the Astrorhizidae. In the more highly developed groups, however, the sessile habit is usually a secondary development from a free-living, benthonic mode of existence. This is illustrated by several of the British Carboniferous genera, where the relationships of these attached forms have been determined by morphological and phylogenetic study.

The change from free, benthonic activity to a static mode of existence leads to rapid, and often extensive, alteration of form, during the evolution of the stock. The resulting morphological dissimilarity, between the final expression of attached growth and the ancestral condition, frequently creates difficulties in classification. This is illustrated in the development of *Fourstonella* Cummings, 1955, from *Valvulinella* Schubert, 1907, where the apparent morphological dissimilarity is so great that it all but masks the phylogenetic link.

Morphological classification of attached forms is also rendered difficult, in many instances, by the fact that the stringencies and demands of the fixed mode frequently lead to a high degree of isomorphism. In this way derivatives of vastly different ancestral stocks achieve a remarkable similarity of form within the same biome. An illustration of such analogous homeomorphy is seen in *Calcitornella* Cushman and Waters, 1928, and *Nubeculinella* Cushman, 1929.

A further difficulty in the classification of attached, Upper Paleozoic Foraminifera lies in the similarity of form, and symbiotic relationships, that existed between these Foraminifera and certain calcareous algae. Johnson (1950) makes mention of this in his description of a Permian algal-foraminiferal consortium.

In the original description of the Foraminifera of the British Carboniferous and Permian, Brady (1876) grouped most of the forms having an attached mode of life within one genus, *Stacheia*. Current revision of these faunas shows that such forms are referable to several genera, including one hitherto undescribed.

The writer would like to acknowledge the continued support of Prof. Neville George in this research and the valued co-operation and assistance from Dr. Alfred R. Loeblich, Jr., Dr. Helen Tappan, and other American experts and friends.

Family Opthalmidiidae

Subfamily Nubeculariinae

Stacheoides, n. gen.

Stacheia (pars) Brady, 1876, *et alios.*

Type species (here designated): *Stacheia polytrematoides* Brady, 1876.

Description: Test usually attached but free in rare cases, relatively large, of irregular form dependent upon the nature of host; irregular, and often reticulate, growth developing on a tubular pattern with numerous chamberlets; partitions separating chamberlets of differing thickness to those of roofs and floors; wall composed of granules of calcite bound by calcareous cement, but with varying, though small, proportion of adventitious material, usually quartz grains; small, circular apertures present on the apices of the irregularly scattered protuberances.

In thin section, representatives of *Stacheoides* may be identified by their characteristic form and habit, the differing thicknesses of chamberlet partition and floor, etc. (see Fig. 1).

Comparison and affinities: There seems little doubt that this new genus should be referred to the Ophthalmidiidae on the grounds of its close morphological similarity to many genera of that family. The presence of adventitious material, within the wall structure, might indicate an affinity to the Trochamminidae, or to the Miliolidae. *Stacheoides*, however, possesses neither the trochoid form or apertural characters of the former nor the basic wall structure that typifies the latter. The correctness of its inclusion within the Ophthalmidiidae is emphasized by its morphological similarity to *Nubecularia* of the Nubeculariinae, with which it is included.

Nubecularia and *Stacheoides*, as well as being structurally alike, are contemporaneous and have been found occurring together in British Lower Carboniferous deposits. The new genus is distinguished by its chamberal form, ramifying and acervuline mode of growth, and its distinct, apertural features. It may represent an aberrant specialization from *Nubecularia* in the Lower Carboniferous, along morphogenetic lines similar to those on which *Sinzowella* Cushman, 1933, developed from *Nubecularia* in the Tertiary.

In his original description of *Stacheia polytrematoides* Brady (1876) noted that the form differed in many respects from the other species which he included in the genus *Stacheia*. The type species of the latter was designated by Cushman (1927) as *Stacheia marginulinoides* Brady. Hence the fundamental differences of *Stacheia* and *Stacheoides* are reflected in those of the two species, *Stacheia marginulinoides* Brady and *Stacheoides polytrematoides* (Brady). *Stacheia* s.s. is characterized by a wall composed only of granules of calcite bound by calcareous cement, whereas

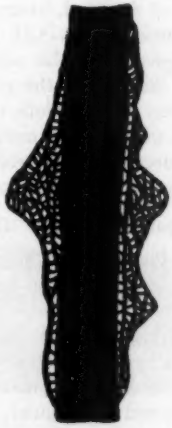

FIG. 1.—*Stacheoides* sp. Diagram based on actual specimen to show typical appearance in thin section. × 20.

Stacheoides has a varying but small amount of incorporated, adventitious material, in addition to the same basic structure. In *Stacheia* s.s. the walls, roofs, and floors of the chamberlets have a constant thickness, which contrasts with the variation in the thickness of the structural elements of *Stacheoides*. The latter possesses a tubular mode of growth which is distinct from the sheetlike formation of chamberlets in *Stacheia*. While the aperture of *Stacheia* s.s. is hidden, single, terminal, and basal, *Stacheoides* has numerous, small apertures, each at the apex of a protuberance.

The distinction of *Stacheoides* and *Fourstonella* Cummings, 1955, is based on the sheetlike habit of growth of the latter.

Preservation and matrix: The normal mode of preservation in argillaceous sediments shows a slight degree of recrystallization of the wall structure. More rarely specimens have been noted which have undergone partial, or complete, silicification. In the limestone facies the degree of alteration is small.

While the protuberances are partly damaged in many specimens, there is little doubt of the distinct character of the apertures when these are examined in well-preserved material.

Horizon and facies: This genus has been recorded throughout the greater part of the British Lower Carboniferous and is also present in Namurian sediments. It is particularly common in the encrinital facies of the Upper Visean of the Midland Valley of Scotland, where it appears to exhibit a commensal habit with the extensive crinoid populations. While this commensalism is well-marked in *Stacheoides*, the genus does not exhibit in any way the symbiotic relationship to calcareous algae that is so common in the case of *Nubecularia*. Indeed, it would seem that *Stacheoides* did not coexist with such algae.

Stacheoides polytrematoides (Brady), 1876

Figs. 2, 3, 7, 8.

Stacheia polytrematoides (*pars*) Brady, 1876, *et alios*.

Description: Test adherent, fusiform outline when attached to spines or polyzoans, irregularly spreading and encrustating on crinoid ossicles; growth characterized by irregular, tubular habit, with numerous, small, uniserial chamberlets forming in rows which ramify over one another and build up into hummocky protuberances at irregularly spaced intervals; chamberlets roughly subcylindrical in form, and subrectangular to square in section; sutures absent but infrequent growth ridges visible; surface often rough and mammillated; wall composed of granular calcite with calcareous cement and a small amount of adventitious material, often forming a thin and incomplete external layer; distinct, small, circular apertures, often in depressions, at the apices of the irregularly scattered protuberances.

Depository, etc.: Lectotype (slide P.35405) in the Brady Collection of Carboniferous and Permian Foraminifera, British Museum (Natural History), from the Hosie Limestone, Hairmyres, Lanarkshire, in the Lower Limestone Group of the Scottish Lower Carboniferous. Figured paratypes (slides P.1015/6/7) in the Protozoa Collection of the Geology Department, University of Glasgow; and slide P.11307 in the Reserve and Study Collection, H.M. Geological Survey, Scottish Office, Edinburgh.

Dimensions: Dependent on the host and hence not confined to type material: maximum length on crinoid ossicle 6.5 mm; maximum length on spine 2.3 mm; maximum breadth on spine 0.67 mm; average width of protuberance 0.17 mm.

Comparison and affinities: This species is readily distinguished from *Stacheia congesta* Brady by the form of growth and apertural characters. It differs from *Stacheoides papillata*, n. sp., in general form and the number, and shape, of the protuberances.

Horizon and facies: Occurring throughout the greater part of the British Lower Carboniferous, this species is also particularly common in the Namurian of the Scottish Midland Valley, in both argillaceous and encrinital facies.

Stacheoides papillata, n. sp.

Figs. 4–6

Description: Test usually attached, rarely free; form dependent upon shape of host, usually fusiform, but may be cylindrical or spherical; surface characterized by numerous, lobiferate, conical to subconical, relatively small protuberances; internally composed of tubular lines of chamberlets, irregularly laid on top of one another and surrounding host, with partitions thinner than roofs or floors; sutures absent but growth ridges and overfolding present; surface smooth; wall composed largely of calcareous grains bound by calcareous cement, but with a small amount of adventitious material included; small, circular, apertural openings within depressions at apices of protuberances.

Depository, etc.: Holotype (slide 9965) and paratype thin section (slide 11306) in the Reserve and Study Collection, H.M. Geological Survey, Scottish Office, Edinburgh, from limestone near the top of the Calciferous Sandstone Series, Lower Carboniferous, of Penton Linn, Dumfrieshire, Scotland.

Figured specimen (slide 11305) in the Reserve and Study Collection, etc., from a limestone near the top of the Calciferous Sandstone Series, Lower Carboniferous, on the left bank of the Liddel Water, Dumfrieshire, Scotland.

Dimensions: Largely dependent on host: Holotype: length 1.34 mm, breadth 0.7 mm. Paratype: average internal length of chamberlet 0.05 mm.

Comparison and affinities: Though similar to

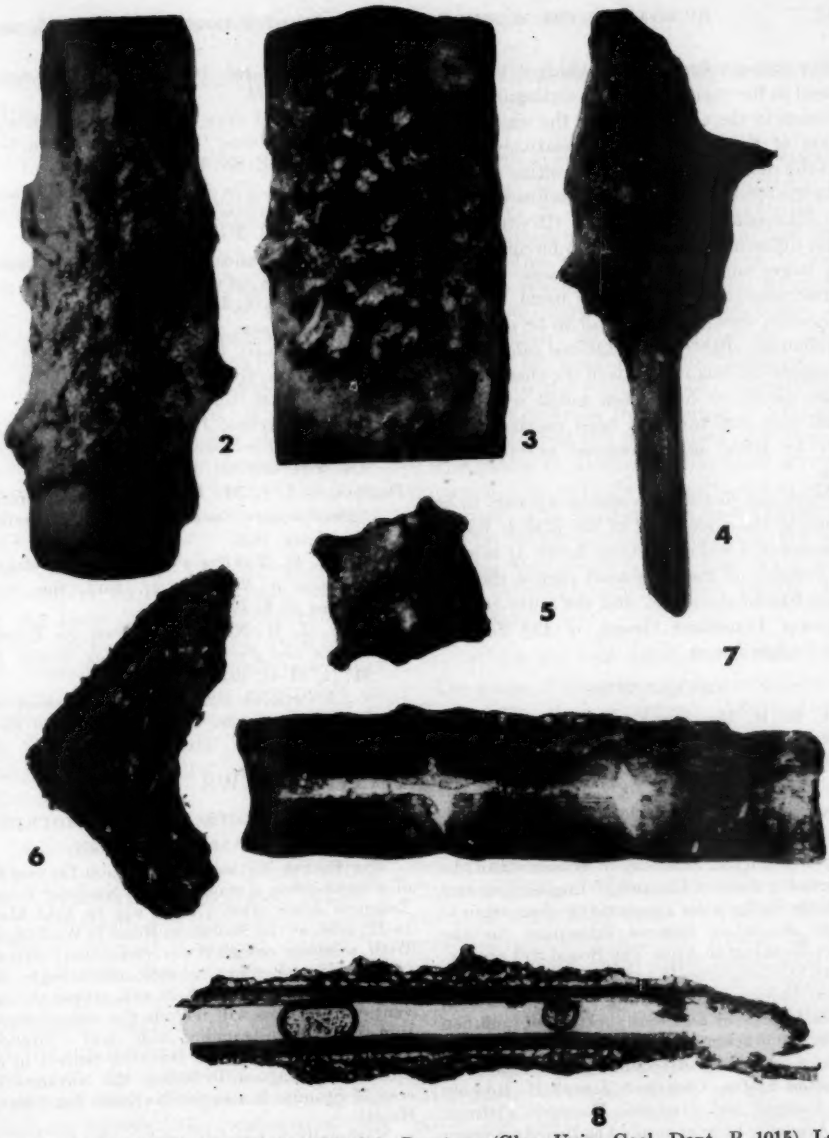

FIG. 2.—*Stacheoides polytrematoides* (Brady): Paratype (Glas. Univ. Geol. Dept. P. 1015). Lateral view of specimen on crinoid ossicle. × 20.

FIG. 3.—*Stacheoides polytrematoides* (Brady): Paratype (Glas. Univ. Geol. Dept. P. 1016). Lateral view of specimen on crinoid ossicle. × 11.

FIG. 4.—*Stacheoides papillata*, n. sp.: Figured specimen (Geol. Surv. Sctld. R. & S. C. 11305). Lateral view of specimen on spine. × 30.

FIG. 5.—*Stacheoides papillata*, n. sp.: Holotype (Geol. Surv. Sctld. R. & S. C. 9965). Lateral view showing apertures. × 33.

FIG. 6.—*Stacheoides papillata*, n. sp.: Paratype (Geol. Surv. Sctld. R. & S. C. 11306). Thin section about equatorial plane. × 32.

FIG. 7.—*Stacheoides polytrematoides* (Brady): Paratype (Glas. Univ. Geol. Dept. P. 1017). Thin section of specimen on crinoid fragment. × 23.

FIG. 8.—*Stacheoides polytrematoides* (Brady): Paratype (Geol. Surv. Sctld. R. & S. C. 11307). Thin section of specimen on spine. × 27.

Stacheia congesta Brady, with which it has been confused in the past, this form is distinguished by differences in the composition of the wall, in the position of the apertures, the relatively larger size of the chamberlets, and the variation in thickness of the structural elements. It is distinguished from *Stacheoides polytrematoides* (Brady) by a general difference in form, and in having a relatively larger number of protuberances.

Preservation and matrix: The usual mode of preservation shows the test wall to be preserved in a yellowish, slightly recrystallized calcite, with clear calcite infilling the voids of the chamberlets.

Rare specimens have been noted where the original wall-structure has been replaced completely by silica, now preserved as crystalline quartz.

Horizon and facies: This species appears to be confined to the upper part of the British Visean and occurs in a wide variety of facies. It is especially common in the uppermost part of the Calciferous Sandstone Series, and the lower part of the Lower Limestone Group, of the Scottish Lower Carboniferous.

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WASHINGTON SCIENTIFIC NEWS

ENGINEERS AND SCIENTISTS PROMOTE SCIENCE EDUCATION

The Washington Academy of Sciences and the District of Columbia Council of Engineering and Architectural Societies announce the formation of a Joint Board on Science Education for the Greater Washington Area. The Board will cooperate with the local schools and laboratories in their common objective of training future scientists. There is at present a serious shortage of qualified engineers and scientists which may well impede the social, health, and technological progress in the United States. Chairman Joseph H. Broome of the Council and President Margaret Pittman of the Academy have appointed as Board members the following persons: Raymond J. Seeger, chairman, National Science Foundation; Henry H. Armsby, Higher Education Division, Department of Health, Education, and Welfare; James F. Fox, Bureau of Ordnance, Department of Navy; W. S. Higginson, Geological Survey; Keith C. Johnson, D. C. Public Schools; Walter H. McCartha, General Services Administration; Martin A. Mason, School of Engineering, George Washington University; W. T. Reed, Department of Defense; and Arnold H. Scott and John K. Taylor, National Bureau of Standards.

SCIENCE TEACHERS SCHEDULE FOURTH NATIONAL CONVENTION

The Fourth National Convention for teachers of science being planned by the National Science Teachers Association (NEA) will be held March 14–17, 1956, at the Shoreham Hotel in Washington. With sessions designed for elementary schools, junior and senior high schools, and colleges, it is expected that 1,500 teachers will attend the convention. Features will include the annual exposition of science-teaching aids and "interview visits" to several of the research centers in and around Washington, including the National Bureau of Standards and the National Institutes of Health.

Nationally known scientists and educators, as well as experienced and successful classroom teachers, will give talks, serve as panel members, and take part as leaders in work discussion groups. The program is being planned especially to give practical helps for classroom teaching situations and problems. The first day's activities will center about the problem of "Learning How to Find Out." This will be followed by the laboratory visits and talks by scientists dealing with "Finding Out What Nobody Knows." The third day of the convention will deal specifically with "Finding Out What We Have Learned." Teacher demonstrations will feature the final day's activities.

PALEONTOLOGY—*New Ordovician echinoderms.* HARRELL L. STRIMPLE, Bartlesville, Okla., et al.

I. THREE NEW GENERA

By HARRELL L. STRIMPLE and WILLIAM T. WATKINS

The latest comprehensive classification of crinoids is that of Moore and Laudon (1943). Considering their definitions of the various subclasses, one has some difficulty in determining the proper placement for one form considered below as *Anthracocrinus*, n. gen.

Under Camerata, p. 76, it is stated, "Crinoids in which all plates of the calyx are united by rigid suture are included in the Camerata." We do not believe the junction of plates in *Anthracocrinus* could be termed "by rigid suture."

Under Flexibilia, pp. 64–65, it is stated, "The Flexibilia comprise dicyclic crinoids having the lower brachials incorporated in the dorsal cup but not rigidly." Such structure is certainly typical of *Anthracocrinus*, which form, however, is not acceptable to the subclass through possession of pinnule bearing arms and five IBB. The arm of the Flexibilia are nonpinnular and there are only three IBB.

Under Inadunata, p. 21, it is stated, "The Inadunata comprises crinoids that have the plates of the calyx joined firmly together, typically by syzygial suture." If our understanding of the terms "joined firmly together" and "syzygial suture" is correct, the definition of the Inadunata requires modification. There are several known forms assigned to the subclass wherein the cup plates were held in place, at least in part, by ligaments. Dr. Moore (1939, pp. 207–208), recognized this condition when he recorded the existence of pits or depressions in the suture edges of the IBB and BB plates of *Plummericrinus mcguirei* (Moore). His explanation of these pits was: "...evidently for reception of ligament fibres." Another example may be obtained by reference to the unretouched illustration of *Allosocrinus bronaughi* Strimple (1949, pl. 4, fig. 4). The right suture edge of a radial plate is exposed therein, and shows a sharp depression in midportion that must have held ligament fibres.

The term syzygial or syzygium in crinoid terminology is usually used in reference to close union of adjacent arm segments or adjacent arms, to the point of almost obliterating the suture. As applied to cup plates, we presume the definition to mean union of cup ossicles by calcified fibers. In Pennsylvanian stratum, thousands upon thousands of isolated crinoid ossicles, as compared to the relatively few dorsal cups, amply demonstrate the absence of syzygial suture much less the firm union of plates.

The plates of *Anthracocrinus* have depressions along the suture edges as found in some inadunates and flexibles and as we presume might be found in some camerates. We are assigning the genus to the Camerata and propose Anthracocrinidae, new family, for its reception.

NOTES ON THE ARCHAEOCRINIDS

The genus *Archaeocrinus* was proposed by Wachsmuth and Springer (1881), with *Glyptocrinus lacunosus* Billings (1857) as the genotype species. The forms involved are rather well known, but some confusion still exists in interpretations of the genus over rather fundamental characteristics. In their description Wachsmuth and Springer state that two plates of the second series are present in all interradial areas with those of the posterior probably a little wider. In the genotype species, three plates follow anal X, and the same is known to occur in several other species. Wachsmuth and Springer (1885) corrected their earlier remarks, concerning the number of plates in the posterior interradius, to three plates in the second series. They gave reference to a communication from W. R. Billings wherein he advised that all the species referred to *Archaeocrinus* by Wachsmuth and Springer (1881), possess a special anal piece placed between the interradials (interbrachials) of the second (first) series. This would include *A. lacunosus* (Billings), *A. marginatus* (Billings), *A. microbasilis* (Billings), and *A. pyriformis* (Billings). One assumes that Billings had material to support such an observation for all the species involved.

It has been noted that a median-ray ridge marks the brachials in the cup and may pass on to the radial plates. The base of the dorsal cup is concave and the infrabasals are confined to the basal concavity with the notable exception of the form originally described as *Thysanocrinus pyriformis* Billings (1857).

In an effort to resolve the status of *T. pyriformis*, the senior author requested Dr. Alice E. Wilson, Geological Survey of Canada (retired), to examine the holotype of the species. She was kind enough to do so, and also to provide a photograph of the specimen, which is numbered 1446b. The proximal portion of the BB are curved slightly inwardly, which has not been shown by Billings or by Wachsmuth and Springer in their illustrations. The specimen figured by Wachsmuth and Springer (1897, pl. 10, fig. 3b), is 1446b; however, it is considerably brightened up and they do not note that it is the holotype. The specimen figured by those authors (pl. 10, fig. 3a), as "the type specimen" is numbered 1446c, and is a paratype. Both specimens demonstrate upflared infrabasal plates that are readily visible in side view of the dorsal cup; therefore, the species is not a bona fide representative of the genus *Archaeocrinus*. This is of particular interest because Moore and Laudon (1943, text-fig. 13), have obviously used the paratype of *Thysanocrinus pyriformis* as the basis for their diagram of the genus *Archaeocrinus*, the type of the family *Archaeocrinidae* Moore and Laudon.

It has been amply demonstrated elsewhere that forms with decidedly upflared IBB are either primitive, or have advanced through stages wherein the base has become concave, with IBB restricted to the concavity, and thence to a stage where the IBB are again upflared. A long evolutionary process could hardly have transpired in these primitive forms and therefore *T. pyriformis* must be considered to be more primitive than *Archaeocrinus*. We consider it desirable to remove the species from *Archaeocrinus*, and since no other genus is suitable for its reception we propose it as the genotype of *Neoarchaeocrinus*, n. gen. On the basis of the existence of upflared IBB, we refer *Archaeocrinus obconicus* Slocum (1924) to *Neoarchaeocrinus*.

We have yet another form to be considered wherein the dorsal cup has the general appearance of *Archaeocrinus*, as restricted, but does not have the characteristic arrangement of plates in the interradius areas. As discussed above, there are two plates in the second series of the interradius areas in all except the posterior interradius, which typically has three. There is at hand a distinctive species from the Bromide of Oklahoma, which has three plates in the second series of all interradius areas with the exception of the posterior which may have either two or three plates in the second series (first interbrachial series). For this form we propose the name *Pararchaeocrinus decoratus*, n. sp., genotype of *Pararchaeocrinus*, n. gen.

Subclass CAMERATA Wachsmuth and Springer

Anthracocrinidae Strimple and Watkins, n. fam.

Dicyclic; cup high; IBB small, elongate, projecting far into the body cavity; RR separated all around; median rays prominent; IBrBr regular, mildly depressed; anal area small, though broader than other interradial areas, median ray absent; proximal brachials and pinnules (ramules?) incorporated into the cup though not always in complete contact; suture edges have depressions, apparently for reception of ligament fibres; lumen large, pentalobate; arms uniserial; tegmen unknown. Range—Ordovician, North America.

Anthracocrinus Strimple and Watkins, n. gen.

The definition of family given above is of course also applicable to the genus. It is noted here that the first interradial plates, including anal X, are in full contact with the basal plates. The bifurcation of arms is significant, $PBrBr_2$ are axillary in all rays and a second branching is present in some half rays, but never in both primary divisions of any one ray. The arms and pinnules do not become free before the second bifurcation.

The most nearly comparable form known to us is *Deocrinus* Hudson (1907). It is readily separable from *Anthracocrinus* in that it does not have a median ridge over the brachials; the large interradial plates are separated from the basals, adjacent radials, and brachials by a series of small plates; and the lumen is round. The meeting of first pinnules of each arm over the interbrachials and the structure of the pinnules and

brachials are remarkably similar for the two genera.

Genotype species.—*Anthracocrinus primitivus* Strimple and Watkins, n. sp.

Anthracocrinus primitivus Strimple and Watkins, n. sp.

Figs. 1a–c, 2a, 4–6

Dorsal cup high with very deep basal concavity. Five elongate IBB and more than half the length of the five BB form a tubular chamber extending far into the body cavity. The proximal columnals are usually in place so that it is necessary to examine this structure from within the cup. A sharp ridge on the BB forms a pentagonal rim about the invaginated base. Five RR are moderately large, pentagonal plates. Five large interradials are in full contact with BB below, RR and BrBr to the lateral sides and with two small interbrachials above. The posterior IR (anal X) is smaller than other interradials and the succeeding plates are larger than found in the other four interrays.

There are typically 3 arms to a ray. First bifurcation takes place with $PBrBr_2$ in all rays. Second bifurcation is usually found on $SBrBr_2$ but in one observed instance occurred on SBr_1. Only one-half ray in any one arm will branch more than once. Usually the second brachial above the second bifurcation in a ray will bear a pinnule. In some instances, dependent upon the presence or absence of IBrBr plates, the third or fourth succeeding brachial bears a pinnule, and the next brachial. The succeeding brachial will normally be nonpinnular, and the next brachial will have a pinnule on the side opposite the first pinnule. Succeeding brachials have one pinnule on opposite sides. The first few plates of the lower pinnules are very large and meet over the two plates of the interrays. There is a loose union between lower segments of the pinnules and brachials that in effect make them part of the dorsal cup and would certainly have prevented any pronounced movement of these elements.

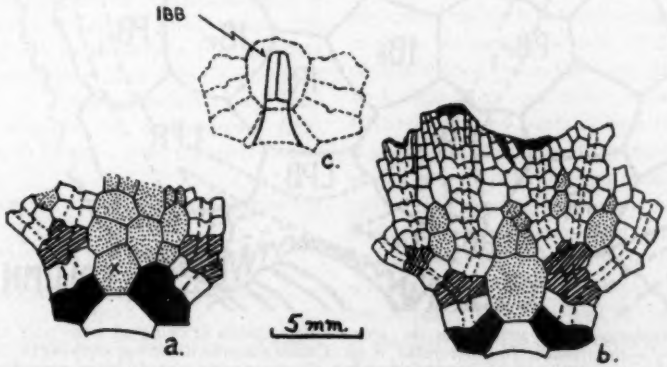
The proximal portion of the stem is large and composed of alternatingly expanded, round segments which do not appear to bear cirri. Very wide, relatively thick columnals are followed by very thin, narrow columnals. Small nodes girdle the mid-section of the thicker columnals. The lumen is large, pentalobate in outline.

The tegmen has not been observed.

Measurements.—As follows:

	Holotype
Width of cup at $PBrBr_2$	10.5 mm.
Height of cup at $PBrBr_2$	3.5 mm
Width of basal cavity	3.2 mm.

Discussion.—This form was discovered by the junior author several years ago at the Spring Creek exposure in the Criner Hills that has also produced *Myeinocystites natus* Strimple (1953a) *Archaeocrinus subovalis* Strimple (1953b) and numerous other unusual forms. Subsequent field work by Allen Graffham and the authors has provided enough material to compile the description given above. One of the main difficulties encountered at the onset was the determination of the nature of and the plates contained in the basal invagination. This difficulty was surmounted when Mr. Graffham stripped off the side of a crown. This is shown in Fig. 1c.



FIGS. 1*a–c*.—*Anthracocrinus primitivus*, n. sp.: Diagrammatic drawings showing (*a*) the posterior interradius, (*b*) normal interradius, and (*c*) placement of infrabasal plates high in the dorsal cup.

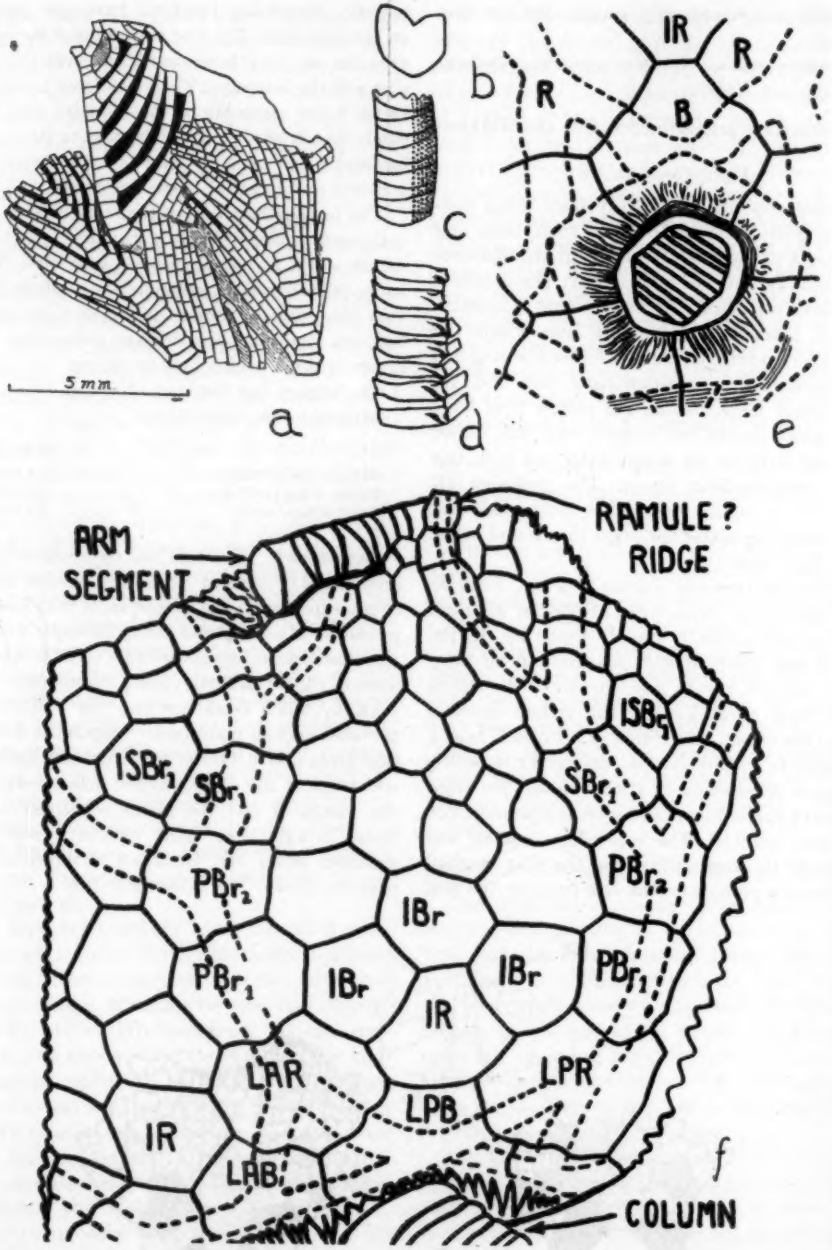


FIG. 2*a*.—*Anthracocrinus primitivus*, n. sp.: Camera-lucida drawing of arms near their termination. FIGS. 2*b–f*.—*Pararchaeocrinus decoratus*, n. sp.: Camera-lucida drawings showing (*b*) a cross section of a brachial plate; (*c*) tegment of arm; (*d*) segment of arm in side view to show pinnular attachment; (*e*) base of cup showing strong crenulation on basal plates and the proximal columnal in place at the bottom of the basal concavity; and (*f*) side view of dorsal cup showing normal interradius and ray ridges (marked by broken lines).

Anthracocrinus primitivus is considerably smaller than associated *Archaeocrinus* and *Pararchaeocrinus*. Specimens are consistently black in color when first found. Washing and brushing will in some instances remove the black coloring. Under mild magnification the plates of the crown are very porous, and it is our thought that a thin skinlike membraneous or leathery covering might have existed in life that is preserved as a black pigment.

The only described species that appears to be closely similar to *Anthracocrinus primitivus* is *Deocrinus asperatus* (Billings) and readily apparent differences have already been given under the comparison of the two genera.

Occurrence.—Lower Bromide formation, Ordovician; east bank of Spring Creek, a tributary of Hickory Creek, Criner Hills, southeast of Ardmore, Okla.

Types.—Holotype and four paratypes to be deposited in the U. S. National Museum.

Family ARCHAEOCRINIDAE Moore and Laudon, 1943

Neoarchaeocrinus Strimple and Watkins, n. gen.

Dorsal cup rather elongated, obconical in shape. Five IBB upflared and readily visible in side view of cup. Five large RR separated all around. Five large BB. Five IR, regular one in each interray with that of the posterior designated as anal X. Two $IBrBr_1$ in all interrays except the posterior which has three, designated as anal plates. Faint median ridges mark the brachials that are incorporated into the dorsal cup for a considerable distance. Arms are cuneiform and relatively small, especially when compared to the size of the dorsal cup.

An excellent diagram of *Neoarchaeocrinus* is given by Moore and Laudon (1943, text-fig. 13) as representative of *Archaeocrinus*. It seems likely that *Neoarchaeocrinus* evolved directly out of the Reteocrinidae. It is closely related to and may be directly ancestrial to *Archaeocrinus* from which it differs in having a more primitive (erect) base.

Genotype species.—*Thysanocrinus pyriformis* Billings.

Range.—Ordovician to lower Silurian.

Pararchaeocrinus Strimple and Watkins, n. gen.

Dorsal cup rather short, subglobular shaped. Five IBB confined to basal concavity. Five large RR separated all around. Five large BB. Each B is followed by a single IR which is in turn followed by three IBrBr with the exception of posterior B. In the posterior interradius there may be two or three anal plates in direct contact with post. B. The posterior interradius is protruded. Each R is followed by a single PBr which is in turn followed by an axillary PBr_2. A small series of IBrBr occur after the bifurcation of the fixed brachials, usually in the arrangement 1–2–3. Strong median ridges mark the brachials. The BB are joined by a confluent ridge that forms a pentagon about the basal concavity. A division of this ridge passes from each B to the adjoining RR so that a stellate shape is formed at their junction at midsection of the RR from whence they join with the ridge marking the brachial plates.

This genus is closely related to *Archaeocrinus* from which it differs mainly in the structure of the plates of the interrays as has been discussed above.

Genotype species.—*Pararchaeocrinus decoratus* Strimple and Watkins, n. sp.

Range.—Ordovician, North America.

Pararchaeocrinus decoratus Strimple and Watkins, n. sp.

Dorsal cup globular, with small basal concavity formed by the proximal edges of the five BB. Five IBB are confined to the base of the concavity. Proximal columnals entirely fill the basal concavity and thereby cover the IBB. Each B is in contact with seven plates with the exception of posterior B, and have raised ridges that pass to adjacent BB forming a pentagonal shaped rim about the basal concavity. Divergent ridges also pass from each B to adjoining RR and converge in midsection of each R thus forming a stellate shaped rim in the lower part of the cup. Post. B is larger and more elongate than other BB and has 8 or 9 facets. A large IR follows each B and is in turn followed by three plates in the second series (IBrBr), except in the posterior interradius. The proximal portion of the posterior interradius is narrower than other interrays, which is partially compensated for through strong protrusion of the cup in this area. The exact number of plates varies, as well as the arrangement of the plates. In the holotype, post. B is followed directly above by a single anal plate (anal X?),

to the right by another anal, and to the right side by yet another anal plate. In the figured paratype, post. B is followed directly above by a single anal plate (anal X?) and to the right side by another anal plate. The only difference in these two specimens is the existence of an extra anal piece above the right anal plate in the holotype that is absent in the figured paratype. The anal plate to the right is followed to the right above by a plate bearing a strong ridge that originates on the right posterior R. The plate is situated too low to be considered in the third series yet is not, sensu stricto, part of the second series. It is followed by a series of hexagonal plates that divide the posterior interray into two parts and carries the raised ridge to the distal termination of the cup. From study of existing material it is difficult to visualize the purpose of this ridge, and series of plates, because the pressure of the gut is apparent in the left portion of the interradius; however, as the raised ridge curves slightly to the left, and the pressure of the gut veers slightly to the right, they become confluent at about midlength of the ridge.

The arms become free on or about the fourth secundibrach and thereafter are narrow, elongated, cuneiform, and bifurcate isotomously at least twice. Each brachial is thin and carries a very thin, extremely elongated pinnule on its thickest lateral side. A special pocketlike arrangement for the reception of the pinnule is shown by text-fig. 2d. Owing to the thinness of the brachial plates, the pinnules are closely spaced. Axillary brachials in the free arms are small and triangular shaped.

The RR are normally seven sided and are followed directly above by six sided, non-axillary first brachials. First bifurcation of rays is with the second pribibrachials, which plates are five sided. The arrangement of intersecundibrachials is 1–2–3.

A significant division of the raised ray ridges takes place with the $SBrBr_2$ wherein a thin ridge passes onto adjoining interbrachials of the large interradius area, and continues to the upper extremity of the cup. There are thus two of these ridges to each interray that continue to the upper edge of the cup where they are only one plate apart. Such a ridge could serve a pinnule and in fact these plates are of course fixed pinnules, but one would expect a ridge on each succeeding fixed pinnule which is not the case. It is more likely that the ridge marks a ramule.

All plates are marked with fine ridges that radiate from the center of each plate and are usually conjunct with those of adjoining ridges. Unusually heavy and erratically shaped, short ridges are found in conjunction with the raised arm rays in the upper portion of the cup.

Proximal columnals are thin, broad ossicles. They are mildly pentagonal in outline and are alternatingly expanded. Text-fig. 2e, shows the proximal columnal in place and the nature of the large lumen. Heavy crenulations are present in the walls of the basal concavity and pass onto the flattened base as illustrated by Fig. 2e, and demonstrates the reason all proximal columnals are usually in place. This arrangement is no doubt matched by similar ridges and depressions on the outer edges of the proximal columnals. The column is rather small considering the size of the crown and the form probably devised this interlocking arrangement to prevent the shock of sudden movement from breaking the crown loose at its proximal extremity where the newer columnals are present. As the columnals became more mature their interlocking crenulations between segments probably sufficed to hold them together.

Measurements.—As follows:

	Holotype, mm.	*Figured paratype, mm.*	*Paratype, mm.*
Width of dorsal cup (approx.)	28.2	22.4	20.0
Height of dorsal cup (approx.)	19.6	15.7	16.0
Width of basal concavity	4.0		4.0
Width of proximal columnals	4.0		4.0
Length of free arms as preserved			39.5

Remarks.—This form was discovered by the junior author several years ago at the Spring Creek exposure of the Bromide formation in the Criner Hills of southern Oklahoma. A colony was excavated by the authors on an expedition to the outcrop in 1948. All specimens were firmly embedded on the underside of slabs and presented a serious problem of preparation for adequate observation. Fortunately another small colony was discovered by Allen Graffham and the senior author several feet from the original colony where the zone approached the surface and the matrix was somewhat disintegrated. Two specimens from the later colony are figured as the holotype and a paratype, and several complete crowns from the first colony are designated as paratypes. One specimen from the Hickory Creek exposure (Rock Crossing) of the Bromide formation, showing the crenulations of the basal concavity and the proximal columnal in place (Fig. 2e), is designated as a paratype.

The relatively common species described as *Archaeocrinus subovalis* Strimple seldom occurs in the same zone with *Pararchaeocrinus decoratus*. One specimen of the former was found on the edge of a slab from the original colony of the latter where the colony was pinching out. Conversely, one specimen of *Pararchaeocrinus decoratus* was found in the large colony of *Archaeocrinus subovalis* that was excavated by us. The next lower zone, and higher zones, carry *Archaeocrinus subovalis* is profusion. The two forms are readily separable on the basis of the arrangement of plates in the interradius areas and due to the unornamented surface of *A. subovalis*.

Types.—Figured holotype, paratype and numerous paratypes are to be deposited in the U. S. National Museum. One slab with several paratypes exposed is to be deposited in the collections of the University of Oklahoma.

REFERENCES

All cited references may be found in Bassler and Moodey, *Bibliographic and Faunal Index of Paleozoic Pelmatozoan Echinoderms*. Geol. Soc. Amer. Spec. Pap. 45, 1943, with the following exceptions:

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——— and LAUDON, L. R. Bull. Geol. Soc. Amer. **46**: 1943.

STRIMPLE, HARRELL L. Bull. Amer. Pal. Soc. **32**: 265–270, pl. 36, fig. 4. 1949.

———. Journ. Washington Acad. Sci. **43**: 105–106. 1953a.

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II. A NEW SPECIES OF CYATHOCYSTIS

By HARRELL L. STRIMPLE and A. ALLEN GRAFFHAM

The discovery of a representative of the rare genus *Cyathocystis* Schmidt (1879) in the lower Bromide formation (Ordovician) of Oklahoma by the junior author warrants considerable attention. Only three species of this edrioasteroid are known. They are *C. plautinae* Schmidt (1880), the genotype, *C. rhizophora* Schmidt (1880), and *C. americanus* Bassler (1936). The first two species are from the Ordovician of Estonia, the later from the Ordovician of Tennessee. The Oklahoma species is described below as *C. oklahomae*, n. sp.

Family CYATHOCYSTIDAE Bather, 1900

Genus **Cyathocystis** Schmidt, 1880

Cyathocystis oklahomae Strimple and Graffham, n. sp.

Figs. 3, 7, 8

Four specimens of the species are available for study, the larger and most perfectly preserved specimen is taken as the holotype. One paratype is perfectly preserved except for the covering plates of the anal opening which are missing in this specimen. The holotype has fine ridges marking the ambulacrals and interambulacrals, and the center sutures between the interlocking ambulacral plates are marked by a well-defined ridge. The smaller specimen does not show this ornamentation so well. There is also evidence of division of the large plates covering the mouth in the holotype that has not been observed in the smaller paratype.

Upper ambulacral surface is bordered by a subpentagonal frame of 31 marginals, excluding the 5 small plates that border the anus. Interambulacrals are five solid plates that are much wider than long. Ambulacra are broad, straight rays with a single series of interlocking covering plates. Five large plates cover the mouth. The anus is situated between a deltoid and the marginal frame, and is covered by a mosaic of small, irregular plates. There is a small shelf on the deltoid bordering the inner edge of the anus.

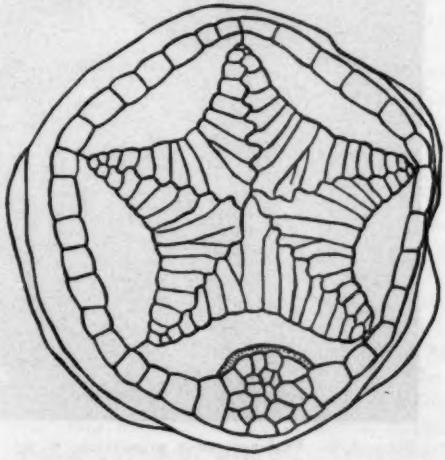


FIG. 3.—*Cyathocystis oklahomae*, n. sp.: Camera-lucida drawing of the oral surface of the holotype.

Aboral portion of the theca is a fused mass of stereom with no visible longitudinal sutures. An encrusting root type of base indicates permanent attachment to a foreign object.

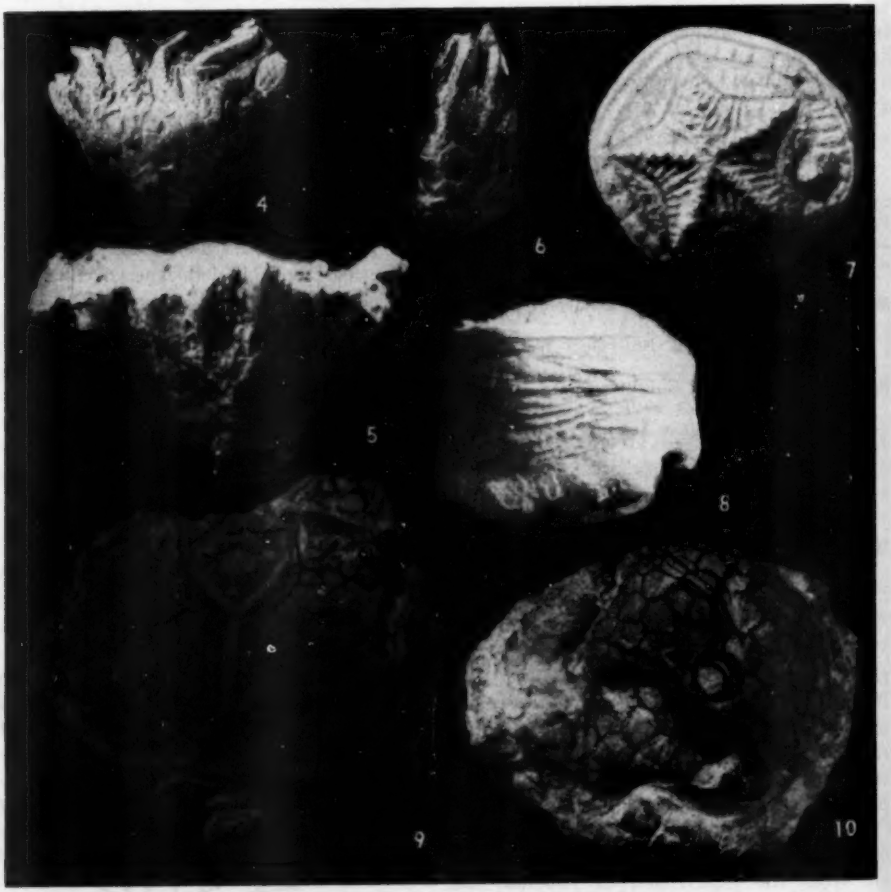
Measurements.—As follows:

	Holotype
Width of theca	7.0 mm.
Height of theca	5.5 mm.
Length of ambulacra	2.8 mm.
Width of deltoid	3.1 mm.
Length of deltoid	0.9 mm.
Greatest width of anus[1]	1.5 mm.

[1] Excluding five plates in line with the marginal plates.

Remarks.—In the genus *Cyathocystis*, the interambulacrals are fused to form five large plates designated as deltoids. The aboral portion of the theca is also fused; however, European species are reported to sometimes disclose obscure longitudinal sutures, irregular in number and position, but never more than five. The anus has five covering plates. There are 40 marginals forming the pentagonal frame bordering the upper ambulacral surface.

C. americanus differs from the European species in having more narrow and elongated ambulacra. The covering plates of the anus are not pre-



FIGS. 4–6.—*Anthracocrinus primitivus*, n. sp.: Unretouched photographs of crowns in side view: 4, A paratype, × 2; 5, the holotype, × 3; a paratype, × 2. FIGS. 7, 8.—*Cyathocystis oklahomae*, n. sp.: Unretouched photographs of the holotype, × 5.3; oral view of theca with anus to the right; side view of theca with anus to the left. FIGS. 9, 10.—*Pararchaeocrinus decoratus*, n. sp.: Side view of a paratype, × 3; the holotype viewed from below, × 2.

served in the holotype, and only known specimen, though the opening is present according to a communication received by the senior author from Dr. Bassler. The deltoids are rather elongated and there appears to be about 40 marginal plates. The theca is elongated and is subpentagonal in outline according to the description of the form. *C. oklahomae* differs from *C. americanus* in having a short, rounded theca, short deltoids, short broad ambulacra, and a reduced number of marginal plates.

C. oklahomae differs from European species in having elongated ambulacral covering plates, reduced number of marginals, shorter deltoids and more than five covering plates for the anus.

Occurrence.—Lower Bromide formation, Ordovician; exposure in the east bank of Spring Creek a tributary of Hickory Creek, Criner Hills, some 7 miles southwest of Ardmore, Okla.

Types.—Holotype and one paratype to be deposited in the U. S. National Museum.

REFERENCES

All references are listed in Bassler and Moodey, *Bibliographic and faunal index of Paleozoic Pelmatozoan* echinoderms, Geol. Soc. Amer. Spec. Pap. 45, p. 199. 1943.

ENTOMOLOGY.—*Three new species of* Culicoides *from Texas (Diptera: Heleidae).*

WILLIS W. WIRTH, Entomology Research Branch, U. S. Department of Agriculture.

(Received July 12, 1955)

In early 1953 it was definitely established that the virus disease of sheep known as bluetongue is present and is occasionally epizootic in the southwestern United States. In South Africa, where bluetongue has caused severe losses to sheep raisers and has been studied intensively for several decades, the only proved vectors are biting midges of the genus *Culicoides*. Drs. D. A. Price and W. T. Hardy, veterinarians of the Texas Agricultural Experiment Station at Sonora, have produced bluetongue infections in sheep experimentally by injections of macerated *Culicoides variipennis* (Coquillett) caught in a light trap on the station where an outbreak of the disease was in course (Journ. Amer. Vet. Med. Assoc. **124:** 255–258. 1954).

In preparation for anticipated further studies on the epidemiology of bluetongue in America, including the determination of the vector species and studies on their biology and control, a survey was begun in May 1953 to determine the distribution of the species of *Culicoides* in the bluetongue area of Texas. Descriptions of three new species taken on the surveys are presented here, in order to make their names available to other workers. The types are deposited in the U. S. National Museum in Washington.

I am greatly indebted to the personnel of the Kerrville laboratory of the Agricultural Research Service for their assistance in the survey.

Culicoides neopulicaris, n.sp.

Fig. 1

♀. Length 1.25 mm, wing 1.13 by 0.5 mm.

Head dark brown, eyes contiguous, bare. Antennae with flagellar segments in proportion of 20:18:18:18:18:18:18:18:20:22:25:28:45, distal sensory tufts on segments 3, 11–15. Palpal segments (Fig. 1, b) in proportion of 10:22:34:13:10 third segment moderately swollen in middle with numerous spoon-shaped sensillae borne on extensive concavity distal to middle of segment.

Mesonotum dark brown, the dorsal surface with yellowish gray pruinosity, with more or less of an indication of a broad median paler gray band from humeral pits to prescutellar sensory depression, the long hairs mixed yellowish and dark brown. Scutellum dark brown, with four strong blackish bristles; pleura very dark brown. Legs uniformly dark brown, becoming somewhat paler on tarsi; hind tibial comb of six long subequal bristles.

Wing (Fig. 1, a) with anterior radial cells complete; costa 0.6 as long as wing. Macrotrichia fairly numerous on distal half of wing and in cell M4 and anal cell. Wing predominantly whitish, the dark markings quite limited, forming essentially three broken transverse bands of spots as figured. The first dark costal spot halfway between wing base and crossvein r-m and extending from costa only across base of media; second costal spot covering distal half of first radial cell, not extending into cell R5; third costal spot just past apex of costa, hourglass-shaped, the

anterior part wider, the posterior part extending only to the fold above vein M1. Small, but quite distinct dark spots also at bases of medial and mediocubital forks, at apices of veins M1, M2, M3–4 and Cu1, the latter two extending on veins M3–4 and Cu1 to their bases, the spot at end of M3–4 extending forward nearly across cell M2 just before tip, and a separate dark spot near apex of cell M1 at level of spot in cell M2. An isolated dark spot in middle of pale area in cell M4 and a dark spot at half the length of anal vein extending back and widening distad into a large dark area along caudal margin extending nearly to vein Cu1. Halteres whitish.

Abdomen dull blackish; spermathecae two, subequal, ovoid, slightly tapered to the ducts.

Male genitalia (Fig. 1, c–d). Ninth sternum with shallow mesal excavation, the posterior membrane bare; ninth tergum distally rounded, with well-developed median lobe, the apicolateral processes practically absent. Basistyles with mesal margins straight, each bearing a dense patch of strong spines towards base, dorsal roots well developed, ventral roots very small; dististyles slender, apices not expanded, slightly incurved. Aedeagus with basal arch rounded, extending to a little more than half of total length, the distal half broad and tapering to a broadly rounded tip. Parameres bent at basal third, the mesal margins approximated on middle third, the apices narrowed to slender, pubescent tips.

Holotype female (type no. 62363, U.S.N.M.), allotype, Kerrville, Tex., June 15, 1953, L. J. Bottimer (light trap). Paratypes: 9 males, 69 females, same data except dates June 13 to October 20, 1953. Other material: 4 females, Ciudad Valles, San Luis Potosí, Mexico, December 1, 1944, B. Brookman (light trap).

This species is closely related to *yukonensis* Hoffman from Alaska and the Yukon, as well as to the Palearctic species *pulicaris* (Linnaeus) and *punctatus* (Meigen), all having, in addition to the pale apex of the second radial cell, a small isolated dark spot in the mediocubital fork. *C. neopulicaris* can be readily separated from the related species, however, by its smaller size, less hairy wings invariably with small but very definite dark spots, and lack of the dark area in cell R5 behind the dark spot over the first radial cell. *C. punctatus* and *yukonensis* differ in having the apices of veins M1 and M2 pale and *punctatus* has a well-developed mesonotal pattern. The obsolete apicolateral processes of the ninth tergite, and the broad apex of the aedeagus will separate the males of *neopulicarus* from those of the other species.

Several males of this species have been received from Dr. Luis Vargas, of the Instituto de Salubridad y Enfermedades Tropicales in Mexico City. They were taken at Chilpancingo in Guerrero, and Dr. Vargas informs me that the species is very abundant in the basin of the River Balsas.

Culicoides bottimeri, n.sp.

Fig. 2

Female—Length 0.9 mm, wing 0.85 by 0.43 mm.

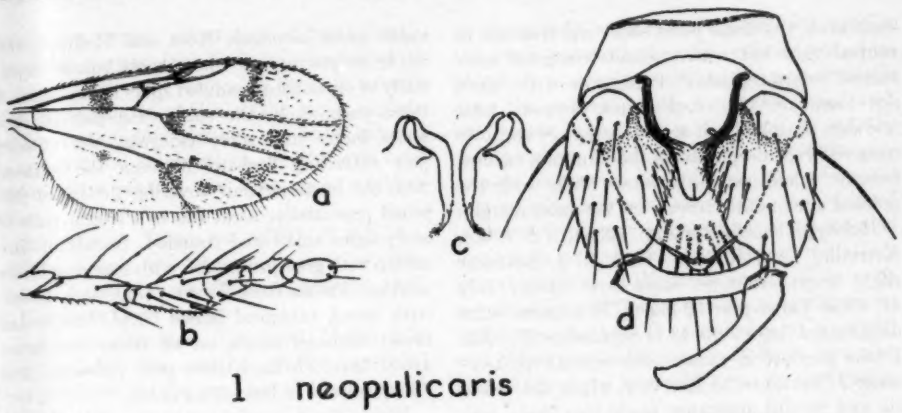
Head dark brown; eyes bare, slightly separated. Antennae with flagellar segments in proportion of 16:11:11:12:12:12:13:13:20:20:20:22:30, distal sensory tufts on segments three to ten inclusive. Palpi (Fig. 2, a) short, segments in proportion of 8:15:35:12:12, third segment markedly swollen, with a broad, shallow, sensory pit.

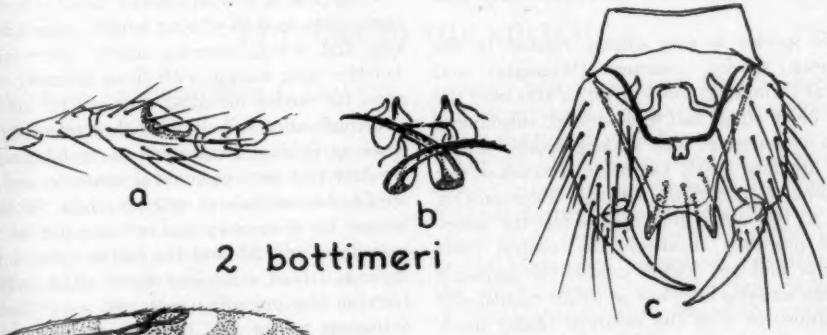
Mesonotum uniformly subshining dark brown, without trace of pruinose spots or vittae, the long and rather numerous brown hairs not restricted to rows. Scutellum dark brown, with two long, submedian, and a few very small, brown hairs. Pleura concolorous with mesonotum. Legs uniformly brownish, slightly paler than thorax, hind tibiae each with five long yellow bristles in apical comb.

Wing uniformly gray, without trace of light or dark spots. Costa ending at 0.53 of wing length; anterior radial cells short and broad, the adjacent radial veins considerably swollen. Wing appearing very hairy, the long macrotrichia dense and extending to wing base behind the anterior media.

Abdomen brownish; spermathecae two, slightly unequal, oval, bases of the ducts not sclerotized.

Male genitalia (Fig. 2, b–c).—Ninth sternum with very shallow, broad, mesal excavation, the posterior membrane spiculate; ninth tergum markedly tapered, with long triangular apicolateral processes, the posterior margin between them semicircular. Basistyles moderately slender and tapering, the dorsal roots short and broad, the ventral roots long and sinuate with broad bases, their apices joined mesad; dististyles slightly incurved and tapering to very slender, pointed tips. Aedeagus with very narrow anterior arms forming a broad and deep, trapezoidal,





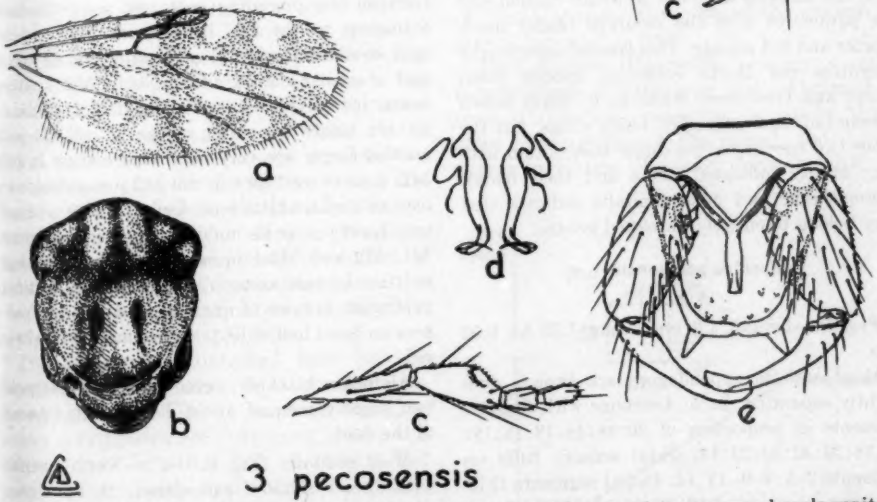


Fig. 1.—*Culicoides neopulicaris:* a, Female wing; b. female palpus; c, male parameres; d, male genitalia, parameres removed.
Fig. 2.—*Culicoides bottimeri:* a, Female palpus; b, male parameres; c, male genitalia, parameres removed.
Fig. 3.—*Culicoides pecosensis:* a, Female wing; b, mesonotal pattern; c, female palpus; d, male parameres; e, male genitalia.
Drawings by Arthur D. Cushman.

basal arch, the distal point short and truncate in ventral view but with a small triangular apex turned ventrocephalad. Parameres with bases not knoblike, the stout stems obtusely bent midway, the distal half of each paramere abruptly recurved ventrocephalad in the form of a curved, heavily sclerotized saberform blade with the pointed apex faintly serrate on the outer margin.

Holotype female (type no. 62364, U.S.N.M.), Kerrville, Tex., June 15, 1953, L. J. Bottimer (light trap). Allotype, same data except July 11, 1953. Paratypes: 12 males, 73 females, same data except dates June 13 to September 27, 1953. I take pleasure in naming this species for Lawrence J. Bottimer, of Kerrville, whose enthusiastic and careful assistance made this study possible.

This species is very closely related to the European species, *cunctans* (Winnertz) and *pumilus* (Winnertz), both of which also have the plain, hairy wings and undecorated, subshining, brown mesonotum. The male genitalia of *bottimeri* are most nearly like those of *cunctans*, but according to Edwards (British Bloodsucking Flies, p. 141, 1939) that species has the apicolateral processes slenderer, the ventral roots slenderer and not joined mesad, the aedeagus with the anterior arch not so broad caudad and the parameres with the recurved blades much shorter and not serrate. This species superficially resembles the North American species *stonei* James and *brookmani* Wirth in its plain brown mesonotum and unmarked hairy wings, but the other two species have a duller mesonotum with very faintly indicated vittae and their female spermathecae and male genitalia indicate that they belong to entirely unrelated groups.

Culicoides pecosensis, n.sp.

Fig. 3

Female.—Length 1.2 mm, wing 1.25 by 0.52 mm.

Head and its appendages dark brown; eyes slightly separated, bare. Antennae with flagellar segments in proportion of 20:18:18:18:18:18: 18:18:30:32:32:32:44, distal sensory tufts on segments 3–5, 7–9, 11–14. Palpal segments (Fig. 3, c) in proportion of 12:36:36:15:15, third segment moderately swollen with a moderately deep, broadly open, sensory pit.

Mesonotum (Fig. 3, b) dark brown with a prominent pattern of pruinose gray spots placed much as in *arboricola* Root and Hoffman, but never so prominent. These spots consist essentially of six pairs of rounded spots in two series of three pairs each, the submedian pair of each series larger and more elongate, the anterior pair extending forward between the humeral pits, the posterior pair covering the broad flattened prescutellar area, the two lateral pairs of each series small and rounded. Scutellum dark brown with gray pruinosity, with four long brown bristles. Pleura dark brown. Legs dark brown, with broad subapical bands on all femora and broad subbasal bands on all tibiae and broad apical band on hind tibia, pale yellowish; 5 or 6 long bristles in hind tibial comb.

Wing (Fig. 3, a) with anterior radial cells complete, costa to 0.55 of wing length; macrotrichia long and dense, covering nearly entire wing. Anterior wing margin with three intensely dark areas the second including apex of first and all of second radial cell. Wing with prominent pale spots as follows: Four spots on costal margin, the first just beyond humeral crossvein and extending beyond base of anterior media, the third beyond tip of second radial cell covering half the breadth of cell R5 and the fourth more or less chevron-shaped with point basad about halfway between the preceding spot and wing tip and extending to the fold before vein M1. A pale spot straddling vein M1 at level of end of costa and a similar one straddling vein M2 a little nearer its apex; a small pale round spot at about its own length from wing margin in cell M1 and another larger one very near wing margin in cell M2; a small oval spot in cell M2 just anterior to base of mediocubital fork. Cell M4 with a broad pale band across its middle half; apices of veins M1, M2 and M3-4 broadly pale margined but no trace of pale area on vein Cu1. Two distinct pale spots in apex of anal cell and a larger pale area on basal half of its posterior margin. Haltere pale.

Abdomen blackish; cerci pale; spermathecae two, slightly unequal, ovoid, very slightly tapered to the ducts.

Male genitalia (Fig. 3, d–e).—Ninth sternite with broad shallow excavation, the posterior membrane bare; ninth tergite with large, triangular apicolateral processes. Basistyles with dorsal and ventral roots subequal, simple and pointed; dististyles with apices slender and incurved. Aedeagus with basal arms nearly straight,

heavily sclerotized, about half of total length, the distal portion broad, slightly expanded in middle, with broadly truncated apex. Parameres with prominent basal knobs, the stems slightly swollen and slightly sinuate, gradually tapered to simple distal points which are abruptly bent laterad, then ventrad and then mesad at their apices.

Holotype ♀ (type no. 62365, U.S.N.M.), allotype, 4 male and 20 female paratypes, Sanderson, Terrell County, Tex., August 29, 1953, H. Brundrett (light trap).

Culicoides arboricola Root and Hoffman is closely related but can be readily separated from *pecosensis* by the presence of the pale spot on the apex of wing vein Cu1, the pale band in cell M4 is narrower, the gray pruinose mesonotal pattern is more prominent and the male genitalia have the aedeagus with a rounded basal arch and longer, very slender distal point and the parameres are more swollen in the middle portion. *C. oursairani* Khalaf lacks the mesonotal pattern, the pale wing spots are more reduced in size and the male aedeagus has a low rounded basal arch and slender distal point, the apicolateral processes of the ninth tergite of the male are slenderer and the parameres are much stouter and more sinuate. *C. guttipennis* (Coquillett) and *villosipennis* Root and Hoffman are also related but have much different male genitalia.

LETTERS TO THE EDITOR

LIMITATIONS ON RAPID SIGNAL ANALYSIS

There are determinable limits to the resolution that can be achieved when a frequency analysis of a signal is made over a brief time interval. The limiting resolution is a function of the least power increment or decrement that can be indicated by the analyzer.

The analyzer should be able to change its indication by at least the smallest detectable power increment during a specified time after sudden onset or removal of the input signal. Thus the rate at which the indication builds up or decays in the analyzer must exceed a limiting value. It is useful to describe this rate by reference to the response time of the analyzer, i.e., the time required to reach substantially steady-state indication after an abrupt change in signal. Because the frequency resolution of an analyzer is directly proportional to the response time, discrimination is sacrificed in the interest of fast response.

The limitations discussed here can be illustrated by reference to a three-dimensional space (see Fig. 1) in which the Cartesian coordinates are frequency, time, and either power or a function of power. The particular problem dealt with here then becomes calculation of a least volume in this space, within which no information about the signal can be found.

The limitations can be illustrated by the familiar properties of a linear series-resonant system, used as a tuned filter to indicate how a particular component of a complex signal fluctuates with time.

Let the observation interval, $\Delta\tau$, be sufficiently large compared with T_0, the undamped natural period of the filter so that phase characteristics can be ignored. Let f_0 be the undamped resonance frequency of the filter. Denote π times the reciprocal of the decrement-per-cycle of the system as Q the "figure of merit."

The smallest discernible change of indicated power is taken as ΔW. Defining a time-attenuation constant α such that, after interruption of the signal, the initial power indication W_0 must decay by $\Delta W >$

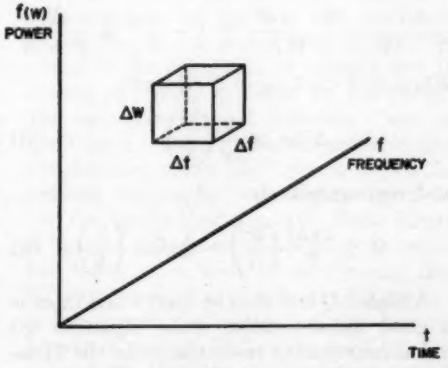


FIG. 1.—Three-dimensional space

$W_0 - W_i = (1 - e^{-\alpha})W_0$ before the change is discerned, then $e^{-(2\pi/Q)(\Delta\tau/T_0)} < e^{-\alpha}$, and

$$Q \leq \frac{2\pi\Delta\tau}{\alpha T_0} = \frac{2\pi f_0 \Delta\tau}{\alpha} \qquad (1)$$

if the filter is to indicate the occurrence of a change in signal during the interval $\Delta\tau$.

Next, consider the relative transmission characteristic of a filter having its Q given by Equation (1) (see Fig. 2). Over a range of frequencies, $\Delta f = f_u - f_l$, the change in filter response does not exceed the smallest detectable power change. The precise result of solving for the discrimination limits in terms of f_u and f_l is

$$\frac{f_u^2 - f_l^2}{f_0^2} > \frac{\alpha}{\pi f_0 \Delta\tau}\sqrt{(e^{\alpha} - 1) + \frac{\alpha^2}{16\pi^2 f_0^2 \Delta\tau^2}} \qquad (2)$$

but if α is relatively small, i.e. fine power discrimination can be made, then:

$$\Delta f \Delta\tau > \frac{\alpha^{3/2}}{2\pi} \qquad (3)$$

An especially interesting expression results from considering the least power change observable to be limited by the noise present with the signal. If S is the signal power and N is the noise power, we take as a limit for detection:

$$\Delta S > N \qquad (4)$$

Considered as a fractional power change:

$$\frac{\Delta S}{S + N} = \frac{W_0 - W_i}{W_0} = 1 - e^{-\alpha} > \frac{N}{S + N}$$

When $S/N \gg 1$, $\alpha \to N/S$ and:

$$\Delta f \Delta\tau > \frac{1}{2\pi}\left(\frac{S}{N}\right)^{-3/2} \qquad (5)$$

and, correspondingly:

$$Q \leq \frac{2\pi\Delta\tau}{T_0}\left(\frac{S}{N}\right) = 2\pi f_0 \Delta\tau \left(\frac{S}{N}\right) \qquad (6)$$

A higher Q can thus be used when there is a good signal-to-noise ratio. Equation (5) has a provocative resemblance to the Heisenberg uncertainty principle. The resemblance is real: both are derived by related mathematics, and both are based on the idea of a least discernible power change.

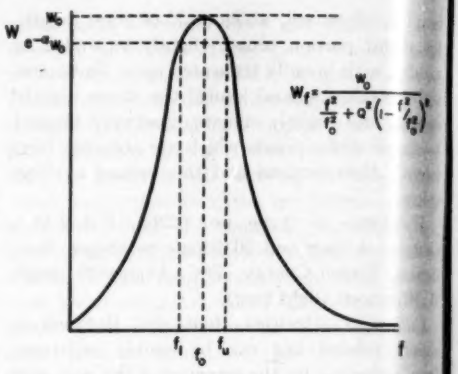


FIG. 2.—Limits of frequency discrimination in filter

A special case of practical importance is the detection of a sinusoidal signal in the presence of "white noise." (Noise energy per unit band-width in cycles/sec. is constant.) For the filter calibrated at its peak response by means of a sinusoidal signal of power W_s, the same indication will be obtained with a "white noise" of E_f units per cycle/sec., given by

$$E_f = \frac{QW_s}{\pi^2 f_0} \qquad (7)$$

This expression is obtained by integrating over the filter response.

Detection of a sinusoidal signal of power S_0 (at the center frequency to which the filter is tuned) in the presence of a noise distribution of n_f units per cycle/sec. proves to require a least time interval $\Delta\tau_0$ which is independent of Q.

$$\Delta\tau_0 \geq \frac{\pi}{2}\frac{n_f}{S_0} \qquad (8)$$

Narrowing the filter does decrease the amount of noise passed, but it lengthens the response time by a proportionate amount.

EDITH L. R. CORLISS
National Bureau of Standards
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